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Returns to growth in a nonparametric DEA approach

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Abstract

In this contribution, first the concept of returns to growth (RTG) of a high-tech firm facing hyper-competition in the new economy is introduced by describing a proportional relationship between growth in inputs and growth in outputs using the growth efficiency (GE) model of Sengupta. Second, both technology- and value-based methods are suggested for estimating the RTG behavior of high-tech firms. Third, although the GE concept seems closely related to the notion of total factor productivity change, this link remains unexplored: we suggest a link between both concepts. Finally, our empirical application to the Indian computer industry reveals that first, companies operating under increasing returns to scale (RTS) may exhibit constant or decreasing RTG; second, companies showing constant RTS may exhibit increasing or decreasing RTG; and third, companies showing decreasing RTS may exhibit constant or increasing RTG. These findings imply that RTS estimates need not provide proper information regarding the growth strategy behavior of high-tech companies.

Keywords: data envelopment analysis; returns to scale; returns to growth; level efficiency; growth efficiency

1. Introduction

In the new economy (internet economy), the dynamic model of hyper-competition is argued to be suitable for examining the efficiency behavior of the technology-intensive industries, such as semiconductors, microcomputers, telecommunications, bioengineering firms, etc. The new economy is differentiated from the old economy in terms of three features: (a) dynamic competition (growth efficiency, GE) as opposed to static competition (static or level efficiency, LE); (b) innovation efficiency, access efficiency, and resource efficiency as opposed to static technical and allocative efficiency; and (c) expanding markets. The growth and decay of these firms are primarily driven by efficiency in both static and dynamic (hyper) competition. While efficiency in

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static competition means the technical (production) and allocative (price) efficiency, efficiency in hyper-competition requires innovation efficiency, access efficiency, and resource efficiency.

Recent times have witnessed that product and process innovations, learning by doing, and network effects have resulted in declining average cost and prices of the high-tech industries, which in turn have resulted in increases in their intrafirm variance in sales and stock market values. Sengupta (2002, 2003, 2004, 2005a,b, 2007, 2011) offers a very detailed account of these phenomena. Also Sengupta and Neogi (2009) discuss various types of policy strategies related to trade policy, information technology (IT) related services, and innovation trends in IT service markets that are all required to capture the *growth* in markets of India's new economy in recent years.

Sengupta (2002) is probably the first author formalizing a model of dynamic economic efficiency to both theoretically illustrate and empirically analyze the growth and decay behavior of firms competing in hyper-competitive markets. He considers a linear data envelopment analysis (DEA) model to characterize the dynamic efficiency frontier in terms of the growth in inputs and a single output. Sengupta and Sahoo (2006) are the first to intensively employ this framework in detail to empirically analyze the GE behavior of the Indian commercial banks and US computer industry. Further developments are found in Sengupta (2003, 2004, 2005a,b, 2007, 2011), among others.

Notice that this budding literature must be distinguished from alternative models specifying the dynamics of production or value functions (e.g. the cost function) in a nonparametric context. For instance, the structure of a dynamic technology is analyzed and nonparametric tests are presented from a dynamic cost-minimizing perspective in Silva and Stefanou (2003). Silva and Stefanou (2007) develop nonparametric dynamic measures of efficiency in the short and the long run in this same framework. Nemoto and Goto (1999) and Ouellette and Yan (2008), among others, develop slight variations on this same theme.

Our first contribution in this study is to introduce the returns to growth (RTG) characterization of a firm in this dynamic efficiency frontier framework. The term RTG is defined as the ratio of the percentage change in the growth in outputs over the percentage change in the growth in inputs. This concept is the dynamic counterpart of the "returns to scale" (RTS) concept defined along the boundary of the static efficiency frontier relating inputs to outputs.

The nature of RTS may be classified under three types: (a) RTS under static DEA, (b) RTS under dynamic DEA, and (c) RTS under GE DEA. First, the static nature of RTS of a firm relates to the proportional increase in all outputs resulting from a given proportionate increase in all inputs in a production technology characterized by a feasible combination of a cross-section of firms in a given time period. The empirical estimation of this type of RTS behavior of a firm has received considerable attention recently (see, e.g., Sueyoshi, 1999; Fukuyama, 2000; Førsund and Hjalmarsson, 2004; Tone and Sahoo, 2003, among others).

Second, the concept of RTS under dynamic DEA setting is fairly recent and describes the same proportional input and output relations in a multistage framework where the black-box nature of technology is extended to two or more stages. In these so-called network models, the intermediate outputs produced in the first stage are considered inputs in the second stage (Sueyoshi and Sekitani, 2005). A variable having the property of being considered both output in the first stage and input in the second stage is called a quasi-fixed input in the literature (Nemoto and Goto, 1999). To our knowledge, no empirical estimation of dynamic RTS appears to have been reported in the literature.

Third, the concept of RTS in a GE setting (i.e. RTG) is completely new to the DEA literature. We develop in this contribution several technology- and value-based methods in DEA framework

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for estimating RTG. Here we empirically illustrate that the informational contents of RTG can be very different from those in static RTS. This finding should caution researchers using the traditional approach of analyzing static RTS behavior that may not be so relevant for high-tech firms facing hyper-competition.

While the GE concept may appear to be closely related to the notion of total factor productivity change (TFPCH), this link has never been explored in the literature. Therefore, a second contribution of this study is to suggest a link between GE and TFPCH.

For the empirical illustration, we analyze a sample of 20 computer hardware companies operating in India over the period 2001–2010 under the conditions of disequilibrium caused by reforms to analyze their GE and RTG performance. The Indian computer hardware market (personal computers, servers, mainframes, workstations, and peripherals) comprises Indian branded players, MNC players, and assembly players.

To introduce efficiency and competition into the industry, the government undertook a series of policy initiatives (including abolition of industrial licensing; simplification of export–import procedures; fiscal policy initiatives concerning the removal of customs duty on IT software, increase of depreciation rate of computers, continuous creation of intellectual capital, that is, scientific and technical manpower on a sustainable basis, etc.). All these policy changes gave rise to a heightened competitive pressure in the industry. The aim of these policy measures was to develop the Indian IT industry as a major global player, and to bring benefits of IT to every walk of life. As a result of these reforms, the Indian computer hardware market has exhibited high (sales) growth in recent years, which can also be attributed partly to the emergence of a large Indian middle class. Thus, the informational contents of various drivers of GE and RTG are not only useful to policy makers in evaluating the outcomes of their reforms, but also to regulators who need to understand and monitor the consequences of their regulations. This analysis enables us to establish a connection between these reforms and GE performance.

The remainder of this paper proceeds as follows. Section 2 offers a discussion of our proposed methodology to estimate RTG behavior of firms, and suggests a linkage between TFPCH and GE. Section 3 illustrates with a numerical example the proposed methodology for the estimation of RTG. Section 4 discusses an empirical application to the Indian hardware computer market for the period 2001-2010. Section 5 contains a few concluding remarks.

2. Methodology

First, we discuss the evaluation of intertemporal technology-based RTG in terms of change in (physical) inputs vis-à-vis (physical) outputs. Thereafter, in an attempt to eliminate the limitations found in technology-based intertemporal RTG, we discuss the evaluation of intertemporal value-based RTG. Second, we propose a link between GE and TFPCH.

2.1. Technology specifications

Assume that we observe n firms where each firm uses m inputs to produce s outputs. For each firm h ($h = 1, 2, \ldots, n$), we denote the input and output vectors for period t by $x_h^t \in R^m$ and

$y^t_h \in R^s$, respectively. Let the input and output price vector be $w^t_h \in R^m$ and $p^t_h \in R^s$, respectively. The observed cost and revenue of firm h are obtained as $c^t_h = \sum_{i=1}^m w^t_{ih} x^t_{ih}$ and $r^t_h = \sum_{r=1}^s p^t_{rh} y^t_{rh}$, respectively. The composite input and output of firm h can be defined as $\hat{x}^t_h = \sum_{i=1}^m s^t_i x^t_{ih}$ and $\hat{y}^t_h = \sum_{r=1}^s s^t_r y^t_{rh}$, where $s^t_i = w^t_{ih} x^t_{ih} / \sum_{i=1}^m w^t_{ih} x^t_{ih}$ and $s^t_r = p^t_{rh} y^t_{rh} / \sum_{r=1}^m p^t_{rh} y^t_{rh}$ are the ith input cost share and rth output revenue share, respectively.

One can define the change in physical inputs and outputs in two alternative ways: either in terms of growth terms, i.e. $(\Delta x^t_{ih}/x^{t-1}_{ih}) = (x^t_{ih} - x^{t-1}_{ih})/x^{t-1}_{ih}$ and $(\Delta y^t_{rh}/y^{t-1}_{rh}) = (y^t_{rh} - y^{t-1}_{rh})/y^{t-1}_{rh}$ or in ratio terms, i.e. (x^t_{ih}/x^{t-1}_{ih}) and (y^t_{ih}/y^{t-1}_{rh}) for all i and r. One can similarly define the growth in cost, revenue, composite input and output in an analogous manner. However, we restrict ourselves in defining the change variables in ratio terms. Notice that defining the change in input and output variables in growth terms can lead to some negative growth values whenever there is a decline in some of these variables over time. However, the treatment of negative values in DEA leads to considerable complications. For instance, one solution is to add a suitable constant translation vector to let all negative growth variables become positive (Seiford and Zhu, 2002), another solution is to apply some variation on the directional distance function model (Portela et al., 2004; Kerstens and Van de Woestyne, 2011; etc.).

2.2. On estimating RTG

2.2.1. Technology-based RTG

Let us define a general intertemporal DEA technology (T) with m inputs and s outputs between any two time periods $t - 1$ and t as follows:

$$T \equiv \left\{ \left(\frac{x^t}{x^{t-1}}, \frac{y^t}{y^{t-1}} \right) \middle| \sum_{j=1}^n \left(\frac{x^t_{ij}}{x^{t-1}_{ij}} \right) \lambda_j \leq \frac{x^t_i}{x^{t-1}_i} (\forall i), \sum_{j=1}^n \left(\frac{y^t_{rj}}{y^{t-1}_{rj}} \right) \lambda_j \geq \frac{y^t_r}{y^{t-1}_r} (\forall r), \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0 \right\}. \tag{1}$$

We also consider some special cases of this technology (1) below (e.g. single output case, revenue as a single "output," etc.). Based on T in (1), we set up the following model (Model I) to measure the *growth* (*dynamic*) efficiency of firm h as:

$$\begin{aligned} &\mathrm{Min}_{\theta,\lambda}\theta \\ &\text{s.t.} \sum_{j=1}^n \left(x^t_{ij}/x^{t-1}_{ij}\right)\lambda_j \leq \left(x^t_{ih}/x^{t-1}_{ih}\right)\theta(\forall i), \quad \sum_{j=1}^n \left(y^t_{rj}/y^{t-1}_{rj}\right)\lambda_j \geq \left(y^t_{rh}/y^{t-1}_{rh}\right)(\forall r), \\ &\sum_{j=1}^n \lambda_j = 1, \quad \lambda_j \geq 0. \end{aligned} \tag{2}$$

If firm h is *growth* (*dynamic*) efficient, i.e. $\theta^* = 1$, then the following intertemporal (*dynamic*) transformation function $F(y_h^t/y_h^{t-1}, x_h^t/x_h^{t-1}) = 0$ for firm h holds:

$$F\left(\frac{y_h^t}{y_h^{t-1}}, \frac{x_h^t}{x_h^{t-1}}\right) = \sum_{r=1}^{s} u_r \left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right) - \sum_{i=1}^{m} v_i \left(\frac{x_{ih}^t}{x_{ih}^{t-1}}\right) + v_o = 0, \tag{3}$$

where v_i, u_r, and v_o are the dual multipliers obtained from the following Lagrange function:

$$L = -\theta + \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n}\left(\frac{y_{rj}^t}{y_{rj}^{t-1}}\right)\lambda_j - \left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right)\right) + \sum_{i=1}^{m} v_i \left(\theta\left(\frac{x_{ih}^t}{x_{ih}^{t-1}}\right) - \sum_{j=1}^{n}\left(\frac{x_{ij}^t}{x_{ij}^{t-1}}\right)\lambda_j\right)$$
$$+ v_o\left(\sum_{j=1}^{n}\lambda_j - 1\right). \tag{4}$$

Following Baumol et al. (1982), one can now derive from (3) the local measure of RTG in production $\varepsilon(y_h^t/y_h^{t-1}, x_h^t/x_h^{t-1})$ for firm h as follows:

$$\varepsilon\left(\frac{y_h^t}{y_h^{t-1}}, \frac{x_h^t}{x_h^{t-1}}\right) = -\sum_{i=1}^{m} \frac{\partial F(.)}{\partial\left(x_{ih}^t/x_{ih}^{t-1}\right)}\left(\frac{x_{ih}^t}{x_{ih}^{t-1}}\right) \Bigg/ \sum_{r=1}^{s} \frac{\partial F(.)}{\partial\left(y_{rh}^t/y_{rh}^{t-1}\right)}\left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right) = \frac{1}{1-v_o}. \tag{5}$$

RTG of firm h are increasing, constant, and decreasing if $\varepsilon(y_h^t/y_h^{t-1}, x_h^t/x_h^{t-1}) > 1 (v_o > 0)$, $\varepsilon(y_h^t/y_h^{t-1}, x_h^t/x_h^{t-1}) = 1 (v_o = 0)$, and $\varepsilon(y_h^t/y_h^{t-1}, x_h^t/x_h^{t-1}) < 1 (v_o < 0)$, respectively. However, as a special case, when technology (T) used in Model I involves just a single output, i.e. $r = 1$, one can measure RTG following Model II as:

$$\varepsilon\left(\frac{y_h^t}{y_h^{t-1}}, \frac{x_h^t}{x_h^{t-1}}\right) = -\sum_{i=1}^{m} \frac{\partial F(.)}{\partial\left(x_{ih}^t/x_{ih}^{t-1}\right)}\left(\frac{x_{ih}^t}{x_{ih}^{t-1}}\right) \Bigg/ \frac{\partial F(.)}{\partial\left(y_h^t/y_h^{t-1}\right)}\left(\frac{y_h^t}{y_h^{t-1}}\right) = \frac{1}{1-v_o}. \tag{6}$$

In this case, one may derive from the dynamic transformation function (3) the *growth* (*dynamic*) efficiency frontier of firm h as

$$\left(\frac{y_h^t}{y_h^{t-1}}\right) = -\frac{v_0}{u} + \sum_{i=1}^{m} \frac{v_i}{u}\left(\frac{x_{ih}^t}{x_{hi}^{t-1}}\right). \tag{7}$$

Another form of GE efficiency frontier of firm h can be derived from (7) as

$$\left(\frac{\Delta y_h^t}{y_h^{t-1}}\right) = \beta_0 + \sum_{i=1}^{m} \beta_i \left(\frac{\Delta x_{ih}^t}{x_{hi}^{t-1}}\right), \tag{8}$$

where $\beta_0 = ((-1 - v_0)/u + \sum_{i=1}^{m}(v_i/u))$ and $\beta_i = (v_i/u)$.

Sengupta (2005a,b) has pointed out that the GE frontier (8) has two nice interpretations. The first interpretation is that the underlying production function is of a logarithmic form: $\ln y_h^t = \beta_0 t + \sum_{i=1}^{m} \beta_i \ln x_{ih}^t$ with a time-varying intercept term. In this case, if $\ln y_h^t$ and $\ln x_h^t$ are nonstationary, the usual least-squares technique fails. If these variables are each first difference stationary, then $\Delta \ln y_h^t$

and $\Delta \ln x_h^t$ are both stationary and ordinary least squares, and corrected ordinary least squares are both applicable with valid t and F tests. Thus, if first difference stationarity holds for the output and input data, then taking the first difference on both sides of logarithmic production function, one obtains (8). The second interpretation of GE frontier (8) is due to Solow (1997) who argued that GE characterizes long-run growth, whereas the LE obtained from absolute input and output data specifies the short-run behavior.

Note that the GE frontier (8) can also be directly derived from model (1) if one substitutes x_{ih}^t with Δx_{ih}^t and y_{rh}^t with Δy_{rh}^t where $\Delta x_{ih}^t = x_{ih}^t - x_{ih}^{t-1}$ and $\Delta y_{rh}^t = y_{rh}^t - y_{rh}^{t-1}$; and β_0 equals $-(v_0/u)$, which can be interpreted as the rate of technical progress and technical regress when $\beta_0 > 0$ and $\beta_0 < 0$, respectively. This approach was adopted by Sengupta (2005a,b) in his GE model. Comparing (7) with (8), the latter suffers from the problem of having potentially negative growth data, leading to efficiency estimation problems in standard DEA models, though it scores better over the former in terms of having two nice interpretations (as stated above).

Note that the notion of RTG is very different from that of RTS, which can be computed for firm h in period t from the following DEA model:

$$
\begin{aligned}
&Min_{\theta,\lambda}\theta \\
&\text{s.t.} \sum_{j=1}^{n} x_{ij}^t \lambda_j \le \theta x_{ih}^t (\forall i), \quad \sum_{j=1}^{n} y_{rj}^t \lambda_j \ge y_{rh}^t (\forall r), \quad \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0 (\forall r).
\end{aligned} \tag{9}
$$

If firm h is *level* efficient in period $t,\theta^* = 1$, then the following (static) transformation function $F_s(y_h^t, x_h^t) = 0$ for firm h holds:

$$
F_s\left(y_h^t, x_h^t\right) = \sum_{r=1}^{s} u_r y_{rh}^t - \sum_{i=1}^{m} v_i x_{ih}^t + v_o = 0, \tag{10}
$$

where u_r,v_i, and v_o are the dual multipliers obtained from the following Lagrange function:

$$
L = -\theta + \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} y_{rj}^t \lambda_j - y_{rh}^t \right) - \sum_{i=1}^{m} v_i \left(\theta x_{ih} - \sum_{j=1}^{n} x_{ij}^t \lambda_j \right) + v_o \left(\sum_{j=1}^{n} \lambda_j - 1 \right). \tag{11}
$$

From (10), one may derive the RTS of firm h in period t as

$$
\varepsilon_s\left(y_h^t, x_h^t\right) = -\sum_{i=1}^{m} \frac{\partial F_s(\cdot)}{\partial x_{ih}^t} x_{ih}^t \Bigg/ \sum_{r=1}^{s} \frac{\partial F_s(\cdot)}{\partial y_{rh}^t} y_{rh}^t = \frac{1}{1 - v_o}. \tag{12}
$$

RTS of firm h are increasing RTS (IRS), constant RTS (CRS), and decreasing RTS (DRS) if $\varepsilon_s(y_h^t, x_h^t) > 1 (v_o > 0)$, $\varepsilon_s(y_h^t, x_h^t) = 1 (v_o = 0) = 1$ and $\varepsilon_s(y_h^t, x_h^t) < 1 (v_o < 0)$, respectively. The RTS evaluation in (12) was employed in several studies, for example, Tone and Sahoo (2004, 2005, 2006), Sengupta and Sahoo (2006), Sahoo et al. (2007), etc.

2.2.2. Value-based RTG

If either physical outputs are not well measured or they are not available, then one can consider revenue (r) as a single output in DEA model (1) in which case we derive RTG of firm h as Model III

$$\varepsilon\left(\frac{r_h^t}{r_h^{t-1}}, \frac{x_h^t}{x_h^{t-1}}\right) = -\sum_{i=1}^{m} \frac{\partial F(\cdot)}{\partial\left(x_{ih}^t/x_{ih}^{t-1}\right)}\left(\frac{x_{ih}^t}{x_{ih}^{t-1}}\right)\Bigg/\frac{\partial F(\cdot)}{\partial\left(r_h^t/r_h^{t-1}\right)}\left(\frac{r_h^t}{r_h^{t-1}}\right) = \frac{1}{1-v_o}. \tag{13}$$

In both the technology-based measures of RTG (i.e. (5) and (6)) and the value-based measure of RTG (i.e.(13)), one assumes the input-ratio-mix to be constant. However, this assumption is too strong to measure any relevant growth effects, because a replicated firm of a given size has no practical and economic meaning in real life unless it is well compared with an actual firm of that size. Therefore, as an alternative, one could first consider a technology involving one input, that is total cost (c) and multiple outputs in an intertemporal DEA setting as in the following Model IV:

$$\begin{aligned}&\mathrm{Min}_{\theta,\lambda}\theta\\ &\text{s.t.} \sum_{j=1}^{n}\left(\frac{c_j^t}{c_j^{t-1}}\right)\lambda_j \le \left(\frac{c_h^t}{c_h^{t-1}}\right)\theta, \quad \sum_{j=1}^{n}\left(\frac{y_{rj}^t}{y_{rj}^{t-1}}\right)\lambda_j \ge \left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right)(\forall r), \quad \sum_{j=1}^{n}\lambda_j = 1, \lambda_j \ge 0.\end{aligned} \tag{14}$$

Model (14) is based on an intertemporal technology set (T) defined as

$$\begin{aligned}T \equiv \Bigg\{ \left(\frac{c^t}{c^{t-1}}, \frac{y^t}{y^{t-1}}\right) \Bigg| \sum_{j=1}^{n}\left(\frac{c_j^t}{c_j^{t-1}}\right)\lambda_j \le \frac{c^t}{c^{t-1}}, \sum_{j=1}^{n}\left(\frac{y_{rj}^t}{y_{rj}^{t-1}}\right)\lambda_j\\ \ge \frac{y^t}{y^{t-1}}(\forall r), \sum_{j=1}^{n}\lambda_j = 1, \lambda_j \ge 0\Bigg\}.\end{aligned} \tag{15}$$

For a growth cost efficient firm h, $\theta^* = 1$, and then the following inter-temporal transformation function $F(y_h^t/y_h^{t-1}, c_h^t/c_h^{t-1}) = 0$ for firm h holds:

$$\sum_{\mathrm{r}=1}^{\mathrm{s}} u_r\left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right) - v\left(\frac{c_h^t}{c_h^{t-1}}\right) + u_0 = 0, \tag{16}$$

where u_r, v, and u_o are the dual multipliers obtained from the following Lagrange function:

$$\begin{aligned}L = -\theta + \sum_{r=1}^{s} u_r\left(\sum_{j=1}^{n}\left(\frac{y_{rj}^t}{y_{rj}^{t-1}}\right)\lambda_j - \left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right)\right) + v\left(\theta\left(\frac{c_h^t}{c_h^{t-1}}\right) - \sum_{j=1}^{n}\left(\frac{c_j^t}{c_j^{t-1}}\right)\lambda_j\right)\\ + u_o\left(\sum_{j=1}^{n}\lambda_j - 1\right).\end{aligned} \tag{17}$$

The growth (dynamic) cost efficiency frontier for firm h is then derived as

$$\left(\frac{c_h^t}{c_h^{t-1}}\right) = \frac{u_0}{v} + \sum_{r=1}^{s} \frac{u_r}{v}\left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right). \tag{18}$$

Alternatively, the growth (dynamic) cost efficiency frontier for firm h can be derived from (18) as:

$$\left(\frac{\Delta c_h^t}{c_h^{t-1}}\right) = \alpha_0 + \sum_{r=1}^{s} \alpha_r \left(\frac{\Delta y_{rh}^t}{y_{rh}^{t-1}}\right), \tag{19}$$

where $\alpha_0 = ((u_0 - v)/v + \sum_{r=1}^{s}(u_r/v))$ and $\alpha_r = (u_r/v)$. Note that this growth cost efficiency frontier (19) can also be directly derived from model (14). Just substitute c_h^t with Δc_h^t and y_{rh}^t with Δy_{rh}^t, where $\Delta c_h^t = c_h^t - c_h^{t-1}$ and $\Delta y_{rh}^t = y_{rh}^t - y_{rh}^{t-1}$, and α_0 equals (u_0/v). The latter can be interpreted as the rate of technical progress and regress when $\alpha_0 < 0$ and $\alpha_0 > 0$, respectively. RTG of firm h, $\varepsilon(y_h^t/y_h^{t-1}, c_h^t/c_h^{t-1})$ can be derived from (16) as:

$$\varepsilon\left(\frac{y_h^t}{y_h^{t-1}}, \frac{c_h^t}{c_h^{t-1}}\right) = -\frac{\partial F(\cdot)}{\partial\left(c_h^t/c_h^{t-1}\right)}\left(\frac{c_h^t}{c_h^{t-1}}\right)\Bigg/\sum_{r=1}^{s}\frac{\partial F(\cdot)}{\partial\left(y_{rh}^t/y_{rh}^{t-1}\right)}\left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right) = \frac{1}{1-u_o}. \tag{20}$$

RTG of firm h are increasing, constant and decreasing if $\varepsilon(y_h, c_h) > 1 (u_o > 0)$, $\varepsilon(y_h, c_h) = 1 (u_o = 0)$ and $\varepsilon(y_h, c_h) < 1 (u_o < 0)$, respectively.

Though the RTG measure defined in (20) is relatively flexible in terms of allowing for a variable input-ratio-mix, it still suffers from problems concerning a constant output-ratio-mix, implying no tradeoff between output ratios, which is counterintuitive. This is because price information generally indicates that the opportunity costs of producing one output rather than another are not the same. Consequently, it might be optimal to expand outputs in nonequal proportions reflecting their differing opportunity costs (Sahoo and Tone, 2009a,b).

Therefore, one could consider measuring RTG in two alternative ways: first one with composite output $(\hat{y})$ as output and cost (c) as input; second one with composite output $(\hat{y})$ as output and composite input $(\hat{x})$as input. In the first case, we set up the following linear programming (LP) model, denoted as Model V:

$$\begin{aligned}&\mathrm{Min}_{\theta,\lambda}\theta\\ &\text{s.t.} \sum_{j=1}^{n}\left(\frac{c_j^t}{c_j^{t-1}}\right)\lambda_j \le \left(\frac{c_h^t}{c_h^{t-1}}\right)\theta, \quad \sum_{j=1}^{n}\left(\frac{\hat{y}_j^t}{\hat{y}_j^{t-1}}\right)\lambda_j \ge \left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}\right), \quad \sum_{j=1}^{n}\lambda_j = 1, \lambda_j \ge 0.\end{aligned} \tag{21}$$

The DEA model (21) is based on an inter-temporal technology set (T) defined as

$$T \equiv \left\{\left(\frac{c^t}{c^{t-1}}, \frac{\hat{y}^t}{\hat{y}^{t-1}}\right)\middle| \sum_{j=1}^{n}\left(\frac{c_j^t}{c_j^{t-1}}\right)\lambda_j \le \frac{c^t}{c^{t-1}}, \sum_{j=1}^{n}\left(\frac{\hat{y}_j^t}{\hat{y}_j^{t-1}}\right)\lambda_j \ge \frac{\hat{y}^t}{\hat{y}^{t-1}}, \sum_{j=1}^{n}\lambda_j = 1, \lambda_j \ge 0\right\}. \tag{22}$$

For a growth cost efficient firm h, $\theta^* = 1$, and then the following inter-temporal (dynamic) transformation function $F(\hat{y}_h^t/\hat{y}_h^{t-1}, c_h^t/c_h^{t-1}) = 0$ holds:

$$F\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}, \frac{c_h^t}{c_h^{t-1}}\right) = u\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}\right) - v\left(\frac{c_h^t}{c_h^{t-1}}\right) + u_0 = 0, \tag{23}$$

where u, v, and u_o are the dual multipliers obtained from the following Lagrange function:

$$L = -\theta + u\left(\sum_{j=1}^{n}\left(\frac{\hat{y}_j^t}{\hat{y}_j^{t-1}}\right)\lambda_j - \left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}\right)\right) + v\left(\theta\left(\frac{c_h^t}{c_h^{t-1}}\right) - \sum_{j=1}^{n}\left(\frac{c_j^t}{c_j^{t-1}}\right)\lambda_j\right) + u_o\left(\sum_{j=1}^{n}\lambda_j - 1\right). \tag{24}$$

The growth cost efficiency frontier is then derived as

$$\left(\frac{c_h^t}{c_h^{t-1}}\right) = \frac{u_0}{v} + \frac{u}{v}\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}\right). \tag{25}$$

Alternatively, the growth cost efficiency frontier for firm h can be derived from (25) as:

$$\left(\frac{\Delta c_h^t}{c_h^{t-1}}\right) = \alpha_0 + \alpha_1\left(\frac{\Delta \hat{y}_h^t}{\hat{y}_h^{t-1}}\right), \tag{26}$$

where $\alpha_0 = (u_0 - v + u)/v$ and $\alpha_1 = (u/v)$. Note that the GE frontier (26) can also be directly derived from DEA model (21) if one substitutes c_h^t with Δc_h^t and $\hat{y}_h^t$ with Δy_h^t where $\Delta c_h^t = c_h^t - c_h^{t-1}$ and $\Delta \hat{y}_h^t = \hat{y}_h^t - \hat{y}_h^{t-1}$; and α_0 equals (u_0/v).

RTG of firm h can now be derived from (23) as:

$$\varepsilon\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}, \frac{c_h^t}{c_h^{t-1}}\right) = -\frac{\partial F(\cdot)}{\partial\left(c_h^t/c_h^{t-1}\right)}\left(\frac{c_h^t}{c_h^{t-1}}\right)\Bigg/\frac{\partial F(\cdot)}{\partial\left(\hat{y}_h^t/\hat{y}_h^{t-1}\right)}\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}\right) = \frac{1}{1-u_o}. \tag{27}$$

In the second case, we simple replace cost (c) with composite input ($\hat{x}$) in (21) to end up with Model VI. This yields the following inter-temporal transformation function:

$$F\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}, \frac{\hat{x}_h^t}{\hat{x}_h^{t-1}}\right) = u\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}\right) - v\left(\frac{\hat{x}_h^t}{\hat{x}_h^{t-1}}\right) + v_o = 0. \tag{28}$$

RTG of firm h can then be derived from (28) as:

$$\varepsilon\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}, \frac{\hat{x}_h^t}{\hat{x}_h^{t-1}}\right) = -\frac{\partial F(\cdot)}{\partial\left(\hat{x}_h^t/\hat{x}_h^{t-1}\right)}\left(\frac{\hat{x}_h^t}{\hat{x}_h^{t-1}}\right)\Bigg/\frac{\partial F(\cdot)}{\partial\left(\hat{y}_h^t/\hat{y}_h^{t-1}\right)}\left(\frac{\hat{y}_h^t}{\hat{y}_h^{t-1}}\right) = \frac{1}{1-v_o}. \tag{29}$$

However, if either physical outputs and inputs are not well measured or their unit prices are not available, then one can consider measuring RTG by considering total cost as input and total revenue as output (Model VII). In this case a simple substitution of composite output ($\hat{y}$) with revenue (r) in (21) leads to the following RTG of firm h:

$$\varepsilon\left(\frac{r_h^t}{r_h^{t-1}}, \frac{c_h^t}{c_h^{t-1}}\right) = -\frac{\partial F(\cdot)}{\partial\left(c_h^t/c_h^{t-1}\right)}\left(\frac{c_h^t}{c_h^{t-1}}\right)\Bigg/\frac{\partial F(\cdot)}{\partial\left(r_h^t/r_h^{t-1}\right)}\left(\frac{r_h^t}{r_h^{t-1}}\right) = \frac{1}{1-v_o} \tag{30}$$

that can be obtained from the following intertemporal transformation function:

$$F\left(\frac{r_h^t}{r_h^{t-1}}, \frac{c_h^t}{c_h^{t-1}}\right) = u\left(\frac{r_h^t}{r_h^{t-1}}\right) - v\left(\frac{c_h^t}{c_h^{t-1}}\right) + v_o = 0. \tag{31}$$

We add two closing remarks. First, note that alternative RTG measures [(5), (6), (13), (20), (27), (29) and (30)] developed in this paper can be applied in any empirical application. The relative success of one over the other depends on whether input and outputs and their unit prices are all well measured. However, when input and output markets are not competitive, and inputs and outputs are not homogeneous, the value-based RTG measures are more meaningful over their technology-based counterparts to capture relevant growth effects.

Second, it is to be noted that in all our RTG measures, we assume unique optimal solutions to exist. However, this needs not be true in any DEA application. Following Tone and Sahoo (2006), the upper and lower bounds of the free dual variables (i.e. u_0,v_0) of the respective models can be obtained to measure upper and lower bounds on the RTG estimates.

For example, in case of model (14) if firm h is cost efficient (i.e. $\theta^* = 1$, and all input and output slacks are zero), then one needs to solve the following LP:

$$\begin{aligned}
&\bar{u}_0^*(\underline{u}_0^*) = \max(\min) u_0 \\
&s.t. \sum_{r=1}^{s}\left(\frac{y_{rj}^t}{y_{rj}^{t-1}}\right)u_r + u_0 \leq \left(\frac{c_j^t}{c_j^{t-1}}\right)(\forall j), \\
&\sum_{r=1}^{s}\left(\frac{y_{rh}^t}{y_{rh}^{t-1}}\right)u_r + u_0 = \left(\frac{c_h^t}{c_h^{t-1}}\right)(\forall j), \quad u_r \geq 0(\forall r), u_0\text{: free.}
\end{aligned} \tag{32}$$

The upper and lower bounds of RTG of firm h, $\varepsilon(y_h, c_h)$ are then obtained as follows:

$$\bar{\varepsilon}(y_h, c_h) = \frac{1}{1-\bar{u}_0^*} \quad \text{and} \quad \underline{\varepsilon}(y_h, c_h) = \frac{1}{1-\underline{u}_0^*}. \tag{33}$$

However, if firm h is inefficient, then we need to project it onto the cost frontier using input slack s^{-*} and output slacks $s_r^{+*}(\forall r)$ as follows:

$$\left(c_h^t/c_h^{t-1}\right)^* \leftarrow \left(c_h^t/c_h^{t-1}\right)\theta^* - s^{-*} \quad \text{and} \quad \left(y_{rh}^t/y_{rh}^{t-1}\right)^* \leftarrow \left(y_{rh}^t/y_{rh}^{t-1}\right) + s_r^{+*}(\forall r),$$

and then solve (32) using these projected data to compute $\bar{\varepsilon}(\underline{\varepsilon})$ using formula (33). In a similar fashion, one can compute lower and upper bounds of RTG for other models.

We now turn to suggest the proof of the link between GE and TFPCH.

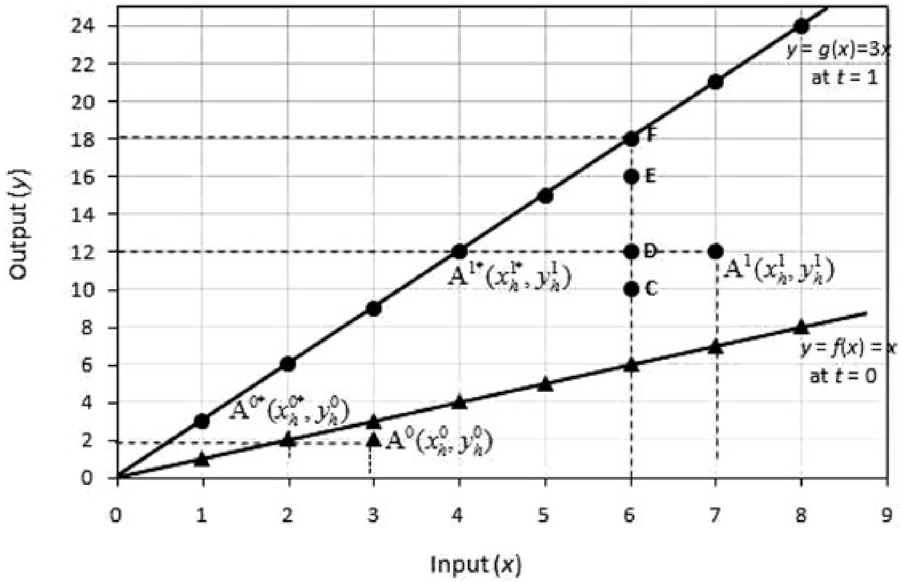

Fig. 1. Production frontiers in two time periods.

2.3. Relationship between TFPCH and GE

Consider in Fig. 1 two constant RTS (CRS) based technology structures in period $t = 0$ and $t = 1$ in which the production frontiers are assumed to be $y^0 = f(x) = \alpha_o x^0$ and $y^1 = g(x) = \alpha_1 x^1$, respectively. Here, α_o and α_1 are assumed to be 1 and 3, respectively. Then, consider a firm h operating inefficiently in both periods, i.e. at point A^0 with (x_h^0, y_h^0) at $t = 0$ and at point A^1 with (x_h^1, y_h^1) at $t = 1$. The total factor productivity change (*TFPCH*) of firm h is then defined as follows:

$$TFPCH_h = \frac{TFP_h^1}{TFP_h^0} = \frac{y_h^1/x_h^1}{y_h^0/x_h^0}. \tag{34}$$

However, by assuming away technical inefficiency in both time periods, one can define *TFPCH* (or equivalently, technical change (*TECHCH*)) of this firm by comparing the ratio of the productivities at two frontier points (i.e. at A^{1*} and A^{0*}) as

$$TECHCH_h = TFPCH_h^* = \frac{TFP_h^{1*}}{TFP_h^{0*}} = \frac{y_h^1/x_h^{1*}}{y_h^0/x_h^{0*}}. \tag{35}$$

Notice that frontier points are denoted by an asterisk (*). Alternatively, we express *TECHCH* analytically as

$$TECHCH_h = TFPCH_h^* = \frac{\alpha_1}{\alpha_0}. \tag{36}$$

An alternative but related measure of "change performance" in the nonparametric literature is the concept of GE, which can be defined by comparing actual input-ratio with minimum input-ratio

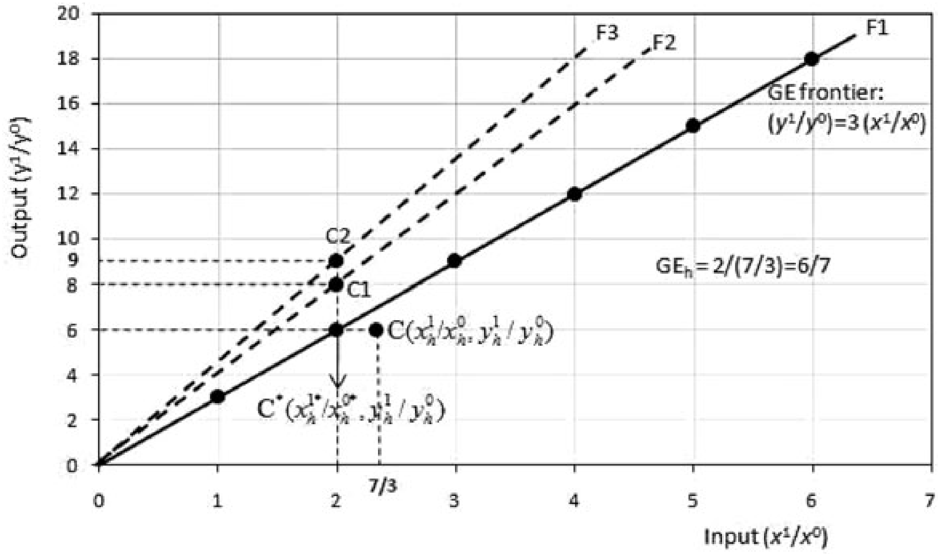

Fig. 2. GE frontier between $t = 0$ and $t = 1$.

for a given output-ratio over the period from $t = 0$ to $t = 1$. The GE frontier ($\overline{0\mathrm{F}1}$) characterized by$(y^1/y^0) = f(x^1/x^0) = (\alpha_1/\alpha_0)(x^1/x^0) = 3(x^1/x^0)$ is drawn in Fig. 2 where firm h is operating inefficiently at point C. (Note that the slope of the GE frontier is based on the assumption that $TFPCH_h$ is no larger than technical change ($TECHCH_h$).)

GE of firm h is then defined from Fig. 2 as

$$GE_h = \frac{\left(x_h^1/x_h^0\right)^*}{\left(x_h^1/x_h^0\right)} = \frac{\left(x_h^{1*}/x_h^{0*}\right)}{\left(x_h^1/x_h^0\right)}. \tag{37}$$

Based on the definition of GE in (37), we have the following proposition:

Proposition 1. For a given (y_h^1/y_h^0), the following equality holds: $(x_h^1/x_h^0)^* = (x_h^{1*}/x_h^{0*})$.

Proof. To produce y_h^0 and y_h^1, the minimum inputs required can be obtained from the base period ($t =$ 0) and current period ($t = 1$) production functions as $x_h^{0*} = (y_h^0/\alpha_0)$ and $x_h^{1*} = (y_h^1/\alpha_1)$, respectively. Therefore, $(x_h^{1*}/x_h^{0*}) = (\alpha_0/\alpha_1)(y_h^1/y_h^0)$. Now, consider the GE frontier. To achieve (y_h^1/y_h^0), the minimum input required is $(x_h^1/x_h^0)^* = (\alpha_0/\alpha_1)(y_h^1/y_h^0)$. Hence, $(x_h^1/x_h^0)^* = (x_h^{1*}/x_h^{0*})$.

The comparison of $TFPCH_h$ and $TFPCH_h^*$ with GE_h clearly reveals some linkage: taking the ratio of $TFPCH_h$ over $TFPCH_h^*$ yields exactly the GE_h expression:

$$\frac{TFPCH_h}{TFPCH_h^*} = \frac{\left(y_h^1/x_h^1\right)/\left(y_h^0/x_h^0\right)}{\left(y_h^1/x_h^{1*}\right)/\left(y_h^0/x_h^{0*}\right)} = \frac{\left(x_h^{1*}/x_h^{0*}\right)}{\left(x_h^1/x_h^0\right)} = GE_h. \tag{38}$$

To numerically illustrate the above proposition, we need to know the coordinates of the following points: $\mathrm{A}^0(x_h^0, y_h^0)$, $\mathrm{A}^{0*}(x_h^{0*}, y_h^0)$, $\mathrm{A}^1(x_h^1, y_h^1)$, $\mathrm{A}^{1*}(x_h^{1*}, y_h^1)$, $\mathrm{C}(x_h^1/x_h^0, y_h^1/y_h^0)$ and $\mathrm{C}^*(x_h^{1*}/x_h^{0*}, y_h^1/y_h^0)$. From Fig. 1, $\mathrm{A}^0(x_h^0, y_h^0) = \mathrm{A}^0(3, 2)$, $\mathrm{A}^{0*}(x_h^{0*}, y_h^0) = \mathrm{A}^{0*}(2, 2)$, $\mathrm{A}^1(x_h^1, y_h^1) = \mathrm{A}^1(7, 12)$, $\mathrm{A}^{1*}(x_h^{1*}, y_h^1)$

$= \mathrm{A}^{1*}(4, 12)$, and hence, $\mathrm{C}(x_h^1/x_h^0, y_h^1/y_h^0) = \mathrm{C}(7/3, 12/2)$ and $\mathrm{C}^*(x_h^{1*}/x_h^{0*}, y_h^1/y_h^0) = \mathrm{C}*(4/2, 12/2)$. Now, one needs to compute the following components: $TFP_h^0 = y_h^0/x_h^0 = 2/3$, $TFP_h^1 = y_h^1/x_h^1 = 12/7$, $TFP_h^{0*} = y_h^0/x_h^{0*} = 2/2 = 1$, and $TFP_h^{1*} = y_h^1/x_h^{1*} = 12/4 = 3$. Hence, $TFPCH_h = \frac{TFP_h^1}{TFP_h^0} = \frac{12/7}{2/3} = \frac{18}{7}$ and $TFPCH_h^* = \frac{TFP_h^{1*}}{TFP_h^{0*}} = \frac{3}{1}$. $GE_h = \frac{(x_h^{1*}/x_h^{0*})}{(x_h^1/x_h^0)} = \frac{4/2}{7/3} = \frac{2}{2.333} = \frac{(x_h^1/x_h^0)^*}{(x_h^1/x_h^0)} = \frac{6}{7}$. Therefore, $\frac{TFPCH_h}{TFPCH_h^*} = \frac{18/7}{3/1} = \frac{6}{7} = GE_h$.

Based on the relationship between GE and $TFPCH$ defined in (38), we have the following proposition.

Proposition 2. $TFPCH_h/TFPCH_h^* = GE_h$ if and only if $TFPCH_h \leq TECHCH_h$, and $TFPCH_h/TFPCH_h^* \neq GE_h$ when $TFPCH_h > TECHCH_h$.

Proof. By definition, $GE_h \leq 1$. This means that

$$\frac{\left(x_h^1/x_h^0\right)^*}{\left(x_h^1/x_h^0\right)} \leq 1$$

$$\text{or,} \frac{\left(x_h^{1*}/x_h^{0*}\right)}{\left(x_h^1/x_h^0\right)} \leq 1$$

$$\text{or,} \frac{\left(x_h^0/x_h^1\right)}{\left(x_h^{0*}/x_h^{1*}\right)} \leq 1$$

$$\text{or,} \frac{\left(y_h^1/y_h^0\right)\left(x_h^0/x_h^1\right)}{\left(y_h^1/y_h^0\right)\left(x_h^{0*}/x_h^{1*}\right)} \leq 1$$

$$\text{or,} \frac{\left(y_h^1/x_h^1\right)/\left(y_h^0/x_h^0\right)}{\left(y_h^1/x_h^{1*}\right)/\left(y_h^0/x_h^{0*}\right)} \leq 1$$

$$\text{or,} \frac{TFPCH_h}{TECHCH_h} \leq 1$$

$$\text{or,} TFPCH_h \leq TECHCH_h$$

To numerically illustrate Proposition 2, we consider the following *three* cases:

Case I: Firm h operates at $A^0(3, 2)$ in $t = 0$ and at $C(6, 10)$ in $t = 1$. Here $TFPCH_h(= 5/2) < TECHCH_h(= 3)$. $TFPCH_h^* = \frac{18/6}{2/2} = 3$. $GE_h(6/3, 10/2) = GE_h(2, 5) = (5/3)/2 = 5/6$, which equals $TFPCH_h/TFPCH_h^* = (5/2)/3$. In this case, the GE frontier ($\overline{0F1}$) is $(y^1/y^0) = 3(x^1/x^0)$.

Case II: Firm h operates at $A^0(3, 2)$ in $t = 0$ and at $D(6, 12)$ in $t = 1$. Here $TFPCH_h(= 3) = TECHCH_h(= 3)$. $TFPCH_h^* = \frac{18/6}{2/2} = 3$. $GE_h(6/3, 12/2) = GE_h(2, 6) = 2/2 = 1$, which equals $TFPCH_h/TFPCH_h^* = 3/3$. In this case, the same GE frontier ($\overline{0F1}$) –$(y^1/y^0) = 3(x^1/x^0)$ holds.

Case III: Firm h operates at $A^0(3, 2)$ in $t = 0$ and at $E(6, 16)$ in $t = 1$. Here $TFPCH_h(= 4) > TECHCH_h(= 3)$. $TFPCH_h^* = \frac{18/6}{2/2} = 3$. $GE_h(6/3, 16/2) = GE_h(2, 8) = 2/2 = 1$, which does not equal $TFPCH_h/TFPCH_h^* = 4/3$, thereby the relationship breaks down. Note that in this case the GE frontier ($\overline{0F2}$) shifts, i.e. $(y^1/y^0) = 4(x^1/x^0)$. However, even if $GE_h < 1$ in which case the GE frontier ($\overline{0F3}$) is determined by a different firm, which is highly inefficient in $t = 0$ but efficient in $t = 1$ (e.g. firm h that operates at $A^0(3, 2)$ in $t = 0$ and at $F(6, 18)$ in $t = 1$ determines the GE frontier as $(y^1/y^0) = 4.5(x^1/x^0)$), the relationship breaks down.

3. A numerical example

Let us consider in Table 1 an example with data consisting of five high-tech firms facing hyper-competition in two different time periods ($t = 0, 1$). Each firm uses single input (x) to produce a single output (y). The change in inputs and outputs of these firms between these two periods is exhibited in the last two columns of this table.

For a visual exposition of the LE vs. GE, and RTS vs. RTG, we exhibit in Figs 3–5 the graphical plots of these data. It is seen from the static production frontier plots in Figs 3 and 4 that the all the five firms are found operating efficiently in period 0. However, only three of them (B, D, and E) are operating efficiently in period 1. Firm A is dubbed inefficient in period 2 because of output slacks. However, in terms of GE frontier over the period, all the firms are found operating efficiently (see Fig. 5). See Table 2 for the LE and GE estimates of these five high-tech firms.

Concerning static RTS behavior, we find in period 0 firm A operating under IRS, firms B and C under CRS, and firms D and E under DRS whereas in period 1 firms A, B, C, and D are under

Table 1
An illustrative data set

	Period 0		Period 1		Period 0–1	
Firms	Input (x^0)	Output (y^0)	Input (x^1)	Output (y^1)	Input (x^1/x^0)	Output (y^1/y^0)
A	2	1	3	4	1.5	4
B	3	3	3	6	1	2
C	4	4	3.5	5	0.875	1.25
D	5	4.5	12	24	2.4	5.333
E	7	5	14	25	2	5

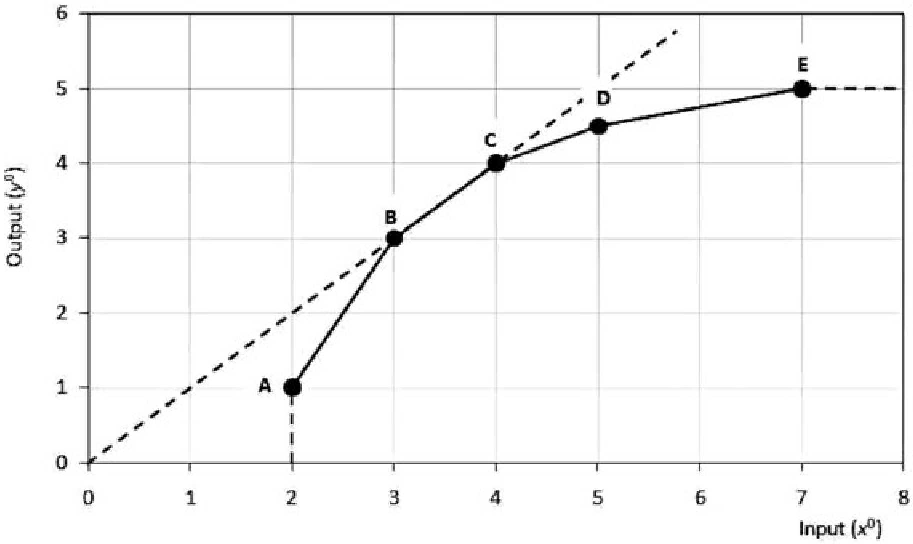

Fig. 3. Static production frontier (Period 0).

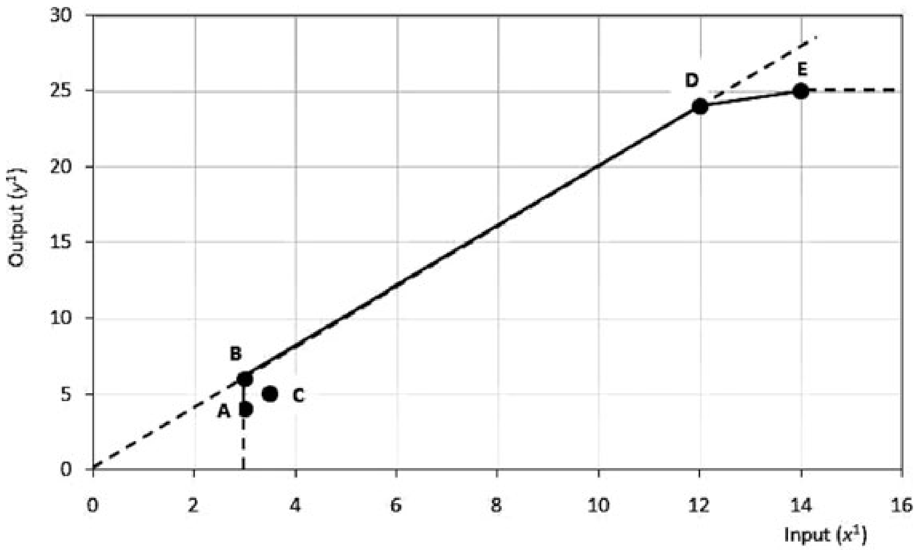

Fig. 4. Static production frontier (Period 1).

CRS and only firm E is under DRS. We clearly see two firms A and D changing their RTS status from period 0 to period 1.

However, in terms of their growth frontier, we find a different perspective on their RTG status. First, firm A that operates under IRS at $t = 0$ and CRS at $t = 1$ exhibits constant RTG, whereas firm D operating under DRS at $t = 0$ and CRS at $t = 1$ exhibits decreasing RTG. Second, though both firms B and C operate under CRS in both periods, they still exhibit increasing RTG. And third, firm E found operating under DRS in both periods exhibits decreasing RTG.

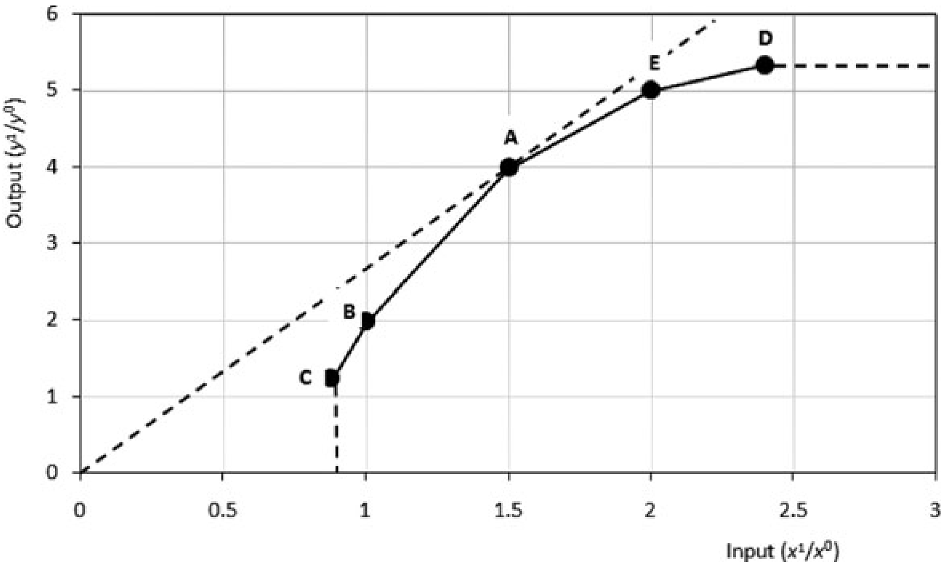

Fig. 5. Dynamic production frontier (Period: 0–1).

Table 2
Estimates of LE vs. GE and RTS vs. RTG

	Period 0				Period 1				Period 0–1			
Firms	LE	RTS(L)	RTS(U)	RTS	LE	SE(L)	SE(U)	RTS	GE	RTG(L)	RTG(U)	RTG
A	1	4	∞	I	1	1	∞	C	1	0.75	1.500	C
B	1	1	2	C	1	1	∞	C	1	2.000	3.000	I
C	1	0.5	1	C	0.857	1	∞	C	1	4.200	∞	I
D	1	0.278	0.556	D	1	0.25	1	C	1	0.000*	0.375	D
E	1	0.000*	0.35	D	1	0.000*	0.28	D	1	0.333	0.8	D

Note: I: Increasing, C: Constant, D: Decreasing.
0.000* = 0.00001.

This example suggests that the static and dynamic estimates concerning efficiency and RTS estimates yield different perspectives with regard to the informational contents on the long-run growth behavior of high-tech firms For example, the static constant RTS estimates of firm B and firm C in both periods indicate that they are mature firms and there are no scale benefits available to grow. However, this is not correct if one views their performance from a GE perspective. The GE frontier reveals that both the firms are growth efficient and have the potential to grow further and survive to capture the available growth benefits as the industry matures. Unless these growth efficient firms grow further, they will turn inefficient and will become unprofitable, and as a result they may leave the industry. This GE model therefore predicts a relation between concentration and efficiency, which is closely related to the traditional survivor technique in the industrial organization literature.

It is therefore imperative that due care must be taken when analyzing the GE and RTG behavior of high-tech firms facing hyper-competition and comparing these to the normal static efficiency and

RTS behavior of traditional firms. This is precisely because diametrically opposite inferences are possible indicating some firms as being mature when they have in fact the potential to grow further and survive to capture additional market share in the industry.

4. An illustrative empirical application

4.1. The data

To empirically illustrate our proposed methodology on the estimation of RTG, we analyze the LE vs. GE and RTS vs. RTG performances of 20 hardware computer companies in India for the period 2001–2010. Two remarks are in order here. First, in the PROWESS database 82 hardware companies are available in total during this 10-year period. However, the selection of only 20 of these 82 companies is made because of their consistent presence in the market throughout the studied period. Some of the input or output data of the remaining 62 companies are unavailable for some years, and therefore could not be considered in this study. Second, despite this missing data problem it is worthwhile pointing out that the time dimension is rather important (i.e. 10 years). Notice that a lot of nonparametric analysis based on, for example the Malmquist productivity index and the like considers much shorter time horizons.

We consider one output—gross sales, and three inputs—manufacturing cost, overhead cost, and maintenance cost. These three cost figures of a company constitute the actual total cost when added together. The Indian computer hardware industry consists of manufacturers of personal computers, servers, mainframes, workstations, and peripherals. Therefore, the companies considered in this industry are heterogeneous because their products are different. The operational data on inputs and outputs of these companies are neither directly comparable nor available. Therefore, we choose financial data to compare their performance. Our sample is limited as to the number of companies, because we consider only those companies whose data are available throughout this 10-year period. These data are obtained from the PROWESS database (complied by the Center for Monitoring Indian Economy (CMIE)).

Tables 3 and 4 present the descriptive statistics of output and input variables used in absolute and growth forms, respectively. Table 4 reveals that all output and input variables have grown fairly steadily over the years. Also evident are the steadily increasing variations in output and input variables, as reflected in their standard deviations becoming ever larger than their means over time. However, the growth patterns of these same output and input variables are found uneven over the years; and there is no clear trend in the nature of the variations in these growth variables as standard deviations and means do not show any clear relationship over years.

We now present our discussion concerning productive performance of the computer hardware companies in terms of their LE and RTS over 10 years.

4.2. LE and RTS

Tables 5 and 6 exhibit the LE and RTS performances, respectively, of hardware computer companies. The average LE performance of the industry is found exhibiting an increasing trend due to the

Table 3
Descriptive statistics of input and output variables (in absolute form)

		2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
(O) Sales (y)	Mean	118.80	114.05	133.71	186.07	187.32	222.94	264.55	283.40	284.84	283.15
	Std. Dev.	100.35	151.25	237.90	345.81	302.44	382.78	459.64	446.90	519.90	511.69
	Min.	3.86	3.38	3.09	3.37	0.57	0.61	0.94	1.1	1.33	0.19
	Max.	336.08	680.56	1,085.52	1,577.90	1,352.86	1,731.9	2,074.6	1,963.24	2,244.38	2,106.37
(I) Manufacturing Cost (x1)	Mean	94.68	86.62	94.61	131.36	150.40	178.49	203.14	228.72	239.38	241.23
	Std. Dev.	76.97	101.22	135.84	211.33	255.23	319.00	345.46	375.57	444.01	434.14
	Min.	2.93	0.32	1.99	2.25	0.84	0.84	0.80	0.75	1.21	0.29
	Max.	231.52	438.46	608.56	958.20	1,157.23	1,456.5	1,551.1	1,613.37	1,858.76	1,681.04
(I) Overhead Cost (x2)	Mean	19.30	18.61	22.05	37.69	38.71	50.72	60.13	69.30	71.96	60.04
	Std. Dev.	19.22	25.35	35.93	72.58	68.12	95.35	121.46	139.02	185.32	153.92
	Min.	0.19	0.24	0.22	0.70	0.31	0.18	0.07	0.21	0.05	0.28
	Max.	55.65	100.96	154.70	321.93	278.99	403.09	529.33	609.15	849.17	704.76
(I) Maintenance Cost (x3)	Mean	0.37	0.26	0.54	0.89	0.73	0.70	0.89	1.13	1.21	0.99
	Std. Dev.	0.43	0.24	1.33	1.99	1.26	1.34	1.83	2.96	3.23	2.68
	Min.	0.01	0.01	0.01	0.03	0.02	0.02	0.02	0.01	0.01	0.01
	Max.	1.65	0.86	6.06	8.46	4.08	5.48	7.63	13.25	14.49	11.96

Note: All the figures are in (Indian) rupees in crores (1 crore = 10 billion).

Table 4
Descriptive statistics of input and output variables (in growth form)

		2001–2002	2002–2003	2003–2004	2004–2005	2005–2006	2006–2007	2007–2008	2008–2009	2009–2010
(O) y (t1)/y (t0)	Mean	0.944	1.177	1.746	1.124	1.135	1.261	1.180	1.089	1.183
	Std. Dev.	0.369	0.838	2.420	0.421	0.379	0.676	0.372	0.614	0.917
	Min.	0.025	0.360	0.108	0.109	0.499	0.439	0.677	0.157	0.041
	Max.	2.025	4.428	11.528	2.294	2.150	3.813	2.025	2.945	4.211
(I) x1 (t1)/x1 (t0)	Mean	0.927	2.933	1.875	1.132	1.130	1.235	1.192	1.086	1.208
	Std. Dev.	0.392	8.474	3.117	0.385	0.373	0.705	0.405	0.648	1.014
	Min.	0.002	0.390	0.182	0.112	0.552	0.428	0.679	0.114	0.093
	Max.	2.099	38.906	14.880	2.091	2.007	3.938	2.200	3.133	4.843
(I) x2 (t1)/x2 (t0)	Mean	0.955	1.205	1.869	0.988	1.329	1.222	2.017	2.322	1.120
	Std. Dev.	0.425	0.765	1.715	0.402	0.634	0.626	2.970	5.445	1.117
	Min.	0.054	0.506	0.538	0.069	0.581	0.003	0.467	0.092	0.134
	Max.	2.178	4.221	7.848	1.825	3.354	2.444	14.317	25.238	5.600
(I) x3 (t1)/x3 (t0)	Mean	1.094	1.612	1.960	1.113	1.160	1.148	1.495	1.102	1.168
	Std. Dev.	0.643	2.373	2.324	0.774	0.835	0.574	2.283	0.447	1.043
	Min.	0.055	0.429	0.364	0.268	0.375	0.235	0.474	0.116	0.281
	Max.	2.500	11.222	8.971	3.696	4.333	2.545	11.000	2.263	5.000

increasing number of fully efficient companies over the years (see Table 5). This finding suggests that the industry is performing better over the years.

With regard to the RTS performance of the industry, the average estimates reveal no clear trend, that is they are of constant, increasing, and decreasing in nature over years. Another interesting

Table 5
Level efficiency performance of computer hardware companies

Companies	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
A C I Infocom Ltd.	1	1	0.985	1	0.976	0.867	0.889	0.888	1	1
Abee Info-Consumables Ltd.	1	1	1	1	1	1	1	1	1	1
Accel Transmatic Ltd.	1	0.787	1	1	1	1	1	1	1	1
C C S Infotech Ltd.	0.993	1	0.977	1	0.877	0.995	0.969	1	1	0.964
C M S Computers Ltd.	0.901	0.904	0.798	0.964	1.000	1	1	1	0.988	1
Compuage Infocom Ltd.	1	1	0.998	1	1	1	1	1	1	1
Computer Point Ltd.	0.883	0.967	0.958	0.983	0.972	1	0.904	1	1	1
Dynacons Systems & Solutions Ltd.	1	0.979	0.904	1	1	0.892	0.940	0.991	0.906	1
Gemini Communication Ltd.	1	0.84	0.792	0.961	1	1	1	1	0.927	0.983
Lipi Data Systems Ltd.	0.891	0.984	0.761	0.886	0.971	0.955	0.959	1	1	1
Moser Baer India Ltd.	1	1	1	1	1	1	1	1	1	1
P C S Technology Ltd.	0.890	0.814	1	1	1	1	1	1	1	1
Saarc Net Ltd.	1	1	1	1	1	1	1	1	1	1
Savex Computers Ltd.	1	1	1	1	1	1	1	1	1	0.940
Smartlink Network Systems Ltd.	0.846	0.900	0.786	0.900	1	1	1	1	0.992	1
T V S Electronics Ltd.	0.825	1	1	0.921	1	0.922	1	0.944	0.940	0.964
V X L Instruments Ltd.	0.822	0.845	0.719	0.814	0.949	0.871	0.849	0.901	1	0.913
Vintron Informatics Ltd.	0.922	0.796	0.657	0.430	1	1	1	1	1	1
X O Infotech Ltd.	0.885	0.773	0.651	0.727	0.786	0.689	1	1	1	1
Zenith Computers Ltd.	1	0.821	0.944	0.984	1	1	0.997	0.959	0.969	0.981
Average	0.943	0.921	0.897	0.928	0.977	0.960	0.975	0.984	0.986	0.987
Std. Dev.	0.068	0.088	0.127	0.139	0.054	0.079	0.045	0.034	0.028	0.024
Min	0.822	0.773	0.651	0.430	0.786	0.689	0.849	0.888	0.906	0.913
Max	1	1	1	1	1	1	1	1	1	1

feature of Table 6 is that the industry is mostly dominated by firms exhibiting CRS followed by firms exhibiting IRS and DRS, respectively.

4.3. *GE and RTG*

Since the performance analysis of high technology-intensive companies on static LE and RTS estimates is less satisfactory, we turn to report their performances based on the estimates GE and RTG.

Table 7 reveals that the average GE estimates exhibit a constant or slightly decreasing trend, and the average GE estimates are comparatively lower compared to the LE estimates reported in Table 5. The low average GE estimates are precisely due to the lesser number of fully growth efficient companies. Regarding the RTG behavior, the industry clearly exhibits increasing RTG, except for 2001–2002. This finding is due to the fact that more companies exhibit increasing RTG followed by firms experiencing constant and decreasing RTG, respectively, over the years.

Comparing LE and GE estimates (see Tables 5 and 7) reveals that companies that are level efficient may not be growth efficient. Let us consider, for example, the case of a few companies. Abee Info-Consumatics attains full LE for all the years, but is found growth inefficient in 2003–2004,

Table 6
Returns to scale performance of computer hardware companies

Companies	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
A C I Infocom Ltd.	0.928	1	1	1	0.990	1.012	1.039	1.022	1.045	1
Abee Info-Consumables Ltd.	1.096	1.269	1.239	1.146	1.070	1.544	1.137	1.099	1.695	2.727
Accel Transmatic Ltd.	1	1.225	1.230	1	1	1	1	1	1	1
C C S Infotech Ltd.	0.999	1	1	1.059	1.075	0.999	1	0.717	1	0.982
C M S Computers Ltd.	1.002	1	1	1	1	1	1	1	1.005	1
Compuage Infocom Ltd.	1	0.897	1	1	0.962	1	1	1	1	1
Computer Point Ltd.	0.990	1	1	1.010	0.973	1	1.024	0.748	1	1
Dynacons Systems & Solutions Ltd.	1	1.054	1	1.014	1	1.014	1.019	0.977	1.040	0.986
Gemini Communication Ltd.	1	1.041	1	1.011	1	1	1	1	0.993	0.985
Lipi Data Systems Ltd.	1	1	1.014	1.022	1.004	0.994	1	1	1	0.950
Moser Baer India Ltd.	1	1	1	1	0.483	0.486	0.494	0.479	0.484	0.492
P C S Technology Ltd.	0.999	1	1	1	1	1	1	1	1	1
Saarc Net Ltd.	1	1	1	1.270	1	1	1	1	1.043	2.574
Savex Computers Ltd.	1	0.560	1	1	1	1	1	0.384	0.244	0.996
Smartlink Network Systems Ltd.	0.998	1.004	1.015	1.002	1	0.948	1	0.918	1.005	0.915
T V S Electronics Ltd.	1	1	1	1	0.831	1	0.981	1.001	1.004	0.929
V X L Instruments Ltd.	1	1	1.029	1.071	1.010	1.026	1.057	0.999	1	0.958
Vintron Informatics Ltd.	1.002	1	1.017	1.110	3.069	2.707	1.698	1.468	1.207	1
X O Infotech Ltd.	1.009	1	1.028	1.195	1.019	1.060	1	1	1	1
Zenith Computers Ltd.	1	1.007	1.013	1.008	0.900	1	0.946	0.920	0.996	0.988
Average	1.001	1.003	1.029	1.046	1.069	1.089	1.020	0.937	0.988	1.124
Std. Dev.	0.028	0.132	0.071	0.077	0.487	0.418	0.200	0.224	0.268	0.534
Min	0.928	0.560	1	1	0.483	0.486	0.494	0.384	0.244	0.492
Max	1.096	1.269	1.239	1.270	3.069	2.707	1.698	1.468	1.695	2.727
No. of companies exhibiting IRS	4	6	8	12	6	6	6	4	8	2
No. of companies exhibiting CRS	11	12	12	8	8	10	11	8	8	8
No. of companies exhibiting DRS	5	2	0	0	6	4	3	8	4	10

2004–2005, and 2008–2009. Accel Transmatic exhibited full LE except for 2002, while it is growth inefficient since 2006. Similarly, Zenith Computers is found level efficient in 2001, and in 2005 and 2006, but it is growth inefficient in the years 2005–2006. Second, in many cases companies that are level inefficient in any two consecutive years and exhibit an improvement in LE are also found to be growth efficient.

On the comparison between RTS and RTG estimates (see Tables 6 and 8), we can make three observations. First, companies exhibiting IRS in both periods may exhibit constant or decreasing RTG (e.g. ACI Infocom, 2007–2009; VXL Instruments, 2003–2004). Second, companies showing CRS in both periods may exhibit increasing or decreasing RTG (e.g. CCS Infotech, 2002–2003; CMS Computers, 2002–2004; Compuage Infocom, 2007–2008). Third, companies exhibiting DRS in both periods may exhibit increasing or constant RTG (e.g. Savex Computers, 2008–2010; Moser Baer India, 2006–2010; Zenith Computers, 2007–2010).

Let us consider the case of a few bigger players in the industry. Zenith Computer exhibits DRS/CRS since 2005 implying there are no scale benefits available to grow further. However, its RTG results indicate growth opportunities to be exploited for future growth. Similarly, while

Table 7
Growth efficiency performance of computer hardware companies

Companies	2001–2002	2002–2003	2003–2004	2004–2005	2005–2006	2006–2007	2007–2008	2008–2009	2009–2010
A C I Infocom Ltd.	1	0.981	0.947	0.912	0.882	1	0.985	1	1
Abee Info-Consumables Ltd.	1	1	0.886	0.855	1	1	1	0.811	1
Accel Transmatic Ltd.	0.997	1	1	1	1	0.733	0.854	0.877	0.781
C C S Infotech Ltd.	1	0.961	0.952	0.913	1	0.808	1	0.950	0.812
C M S Computers Ltd.	0.960	0.922	1	1	1	0.786	1	1	1
Compuage Infocom Ltd.	0.887	1	1	0.951	0.970	0.849	1	0.940	0.915
Computer Point Ltd.	0.971	0.948	0.911	0.915	0.935	1	1	0.976	0.986
Dynacons Systems & Solutions Ltd.	0.884	0.862	1	0.946	0.760	0.861	1	0.824	1
Gemini Communication Ltd.	0.714	1	1	1	1	0.811	0.971	0.797	0.978
Lipi Data Systems Ltd.	1	0.800	1	0.990	1	0.853	0.992	0.846	0.996
Moser Baer India Ltd.	1	1	0.806	0.668	0.865	0.878	0.834	0.864	0.968
P C S Technology Ltd.	0.913	1	0.974	0.914	0.867	0.841	1	0.928	1
Saarc Net Ltd.	1	0.948	0.846	1	1	1	1	1	1
Savex Computers Ltd.	1	1	1	0.925	0.819	1	0.940	0.890	0.924
Smartlink Network Systems Ltd.	0.975	0.928	0.885	0.965	0.829	0.876	0.931	0.915	0.977
T V S Electronics Ltd.	1	1	1	0.983	0.943	0.883	1	0.825	0.916
V X L Instruments Ltd.	1	0.884	1	1	0.902	0.929	0.958	1	0.837
Vintron Informatics Ltd.	0.860	1	1	1	1	1	1	1	1
X O Infotech Ltd.	0.918	0.813	1	0.957	0.820	1	1	1	0.831
Zenith Computers Ltd.	0.862	1	0.928	0.874	0.905	0.746	0.904	0.888	0.939
Average	0.947	0.952	0.957	0.938	0.925	0.893	0.968	0.917	0.943
Std. Dev.	0.075	0.065	0.060	0.078	0.078	0.092	0.051	0.073	0.072
Min.	0.714	0.800	0.806	0.668	0.760	0.733	0.834	0.797	0.781
Max.	1	1	1	1	1	1	1	1	1

TVS Electronics exhibits mostly CRS/DRS since 2004, its RTG estimates reveal increasing growth potential. The same is the case with Moser Baer India that exhibits no scale benefits at all in none of the years of our study period, but which reveals growth opportunities to be exploited in some of these years (2003–2004, 2004–2005, and 2009–2010). These findings suggest that RTS estimates of high-tech companies cannot always provide reliable information concerning whether growth opportunities are available. Therefore, one can resort to the application of GE and RTG methods to seek such information, which are much more relevant for the analysis of high technology-intensive companies facing hyper competition in their market.

5. Concluding remarks

The traditional approach to measuring RTS behavior of high-tech firms facing hyper-competition in a static production frontier model is argued to be theoretically inadequate as it offers contrasting information as to growth strategy behavior of these firms. Therefore, this paper attempts to first introduce the concept of RTG of a high-tech firm using the dynamic GE model of hyper-competition by Sengupta (2002). Then, both technology-and value-based methods are suggested to estimating

Table 8
Returns to growth performance of computer hardware companies

Companies	2001–2002	2002–2003	2003–2004	2004–2005	2005–2006	2006–2007	2007–2008	2008–2009	2009–2010
A C I Infocom Ltd.	1	1.266	1.279	1.032	3.809	1.412	0.902	1	1
Abee Info-Consumables Ltd.	1	1.139	1.246	1.345	2.239	1	1	1	1
Accel Transmatic Ltd.	0.977	2.188	1	1	1	1	1.007	1.077	1.251
C C S Infotech Ltd.	1	1.235	1.065	1.020	1	1.247	1	1	1.094
C M S Computers Ltd.	0.892	1.135	1.022	1	0.499	1	1	1.468	1.188
Compuage Infocom Ltd.	0.970	1.528	1	1.020	0.666	1	0.461	1	1
Computer Point Ltd.	0.912	1.175	1.180	1.029	3.613	1.242	1	1.078	1
Dynacons Systems & Solutions Ltd.	0.890	0.916	1.178	1.023	1.268	1	1	1.082	1
Gemini Communication Ltd.	0.981	1	1	1	0.424	1.202	0.879	1.076	1.023
Lipi Data Systems Ltd.	0.832	1.120	1	1.028	0.571	1.222	1	1	1
Moser Baer India Ltd.	0.482	1	1.017	1.030	0.634	1	1	1	1.066
P C S Technology Ltd.	0.876	1	1.028	1.023	1.235	1	1.825	1.076	1
Saarc Net Ltd.	0.706	1.210	1.241	1	1.330	1	1.105	1	2.934
Savex Computers Ltd.	1	1	1.018	1.024	0.610	1	0.885	1.060	1.026
Smartlink Network Systems Ltd.	0.892	1.137	1.037	1.021	1.286	1.238	1	1.145	1
T V S Electronics Ltd.	1	0.450	0.492	1.021	3.923	1	1.220	1.086	1.026
V X L Instruments Ltd.	1	1.211	1	1.415	4.099	1	1.017	1	1.079
Vintron Informatics Ltd.	0.981	1.340	2.296	1.125	1	1	1	1	1
X O Infotech Ltd.	0.979	1.165	1.056	1.021	2.961	1	2.922	1	1
Zenith Computers Ltd.	0.981	1	1.089	1.026	1.221	1.204	1.175	1.083	1.069
Average	0.918	1.161	1.112	1.060	1.670	1.088	1.120	1.062	1.138
Std. Dev.	0.127	0.320	0.322	0.113	1.273	0.130	0.487	0.106	0.428
Min.	0.482	0.450	0.492	1	0.424	1	0.461	1	1
Max.	1	2.188	2.296	1.415	4.099	1.412	2.922	1.468	2.934
No. of companies exhibiting Increasing RTG	0	13	14	16	11	7	7	10	10
No. of companies exhibiting Constant RTG	6	5	5	4	3	13	9	10	10
No. of companies exhibiting Decreasing RTG	14	2	1	0	6	0	4	0	0

the RTG behavior of high-tech firms. Though GE concept appears to be closely related to the notion of TFPCH, this connection has never been explored in the literature: we suggest a link between these two concepts.

We have empirically illustrated our proposed methodology to analyze the LE vs. GE and RTS vs. RTG performances of 20 hardware computer companies in India for the period 2001–2010. Our main findings reveal that: first, companies that are level efficient may not be growth efficient; second, companies exhibiting IRS in both periods may exhibit constant or decreasing RTG; third, companies showing CRS in both periods may exhibit increasing or decreasing RTG; and fourth, companies exhibiting DRS in both periods may exhibit increasing or constant RTG. Therefore, it is suggested that due care must be taken in analyzing GE and RTG behavior of high-tech firms compared to the static efficiency and RTS behavior of traditional firms, which is precisely because the opportunity cost of doing so may yield diametrically opposite conclusions concerning the growth strategy of firms. Obviously, empirical applications on larger data sets are being called for to deepen our understanding on the comparison between RTS and RTG.

Several potentially promising future research areas can be outlined. One is to compare the dynamic approach initiated by Sengupta (2002) with alternative specifications of dynamic production models like the ones of Silva and Stefanou (2003, 2007), Nemoto and Goto (1999), Ouellette and Yan (2008), among others. Another issue that remains to be explored is the precise nature of the relationship between LE and GE.

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References

Baumol, W.J., Panzar, J.C., Willig, R. D., 1982. *Contestable Markets and the Theory of Industrial Structure*. Harcourt Brace Jovanovich, New York.

Cooper, W.W., Seiford, L.M., Tone, K., 2007. *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*. Kluwer Academic Publishers, Boston, MA.

Färe, R., Grosskopf, S., 1985. A nonparametric cost approach to scale efficiency. *Scandinavian Journal of Economics* 87, 594–604.

Førsund, F.R., Hjalmarsson, L., 2004. Calculating scale elasticity in DEA models. *Journal of Operational Research Society* 55, 1023–1038.

Fukuyama, H., 2000. Returns to scale and scale elasticity in data envelopment analysis. *European Journal of Operational Research* 125, 93–112.

Kerstens, K., Van de Woestyne, I., 2011. Negative data in DEA: A simple proportional distance function approach. *Journal of the Operational Research Society* 62, 1413–1419.

Nemeto, J., Goto, M., 1999. Dynamic data envelopment analysis: Modeling intertemporal behavior of a firm in the presence of productive inefficiencies. *Economics Letters* 64, 51–56.

Ouellette, P., Yan, L., 2008. Investment and dynamic DEA. *Journal of Productivity Analysis* 29, 235–247.

Portela, M. C. A. S., Thanassoulis, E., Simpson, G., 2004. Negative data in DEA: A directional distance approach applied to bank branches. *Journal of the Operational Research Society* 55, 1111–1121.

Sahoo, B.K., Mohapatra, P. K. J., Trivedi, M.L., 1999. A comparative application of data envelopment analysis and frontier translog production function for estimating returns to scale and efficiencies. *International Journal of Systems Science* 30, 379–394.

Sahoo, B.K., Sengupta, J.K., Mandal, A., 2007. Productive performance evaluation of the banking sector in India using data envelopment analysis. *International Journal of Operations Research* 4, 63–79.

Sahoo, B.K., Tone, K., 2009a. Decomposing capacity utilization in data envelopment analysis: An application to banks in India. *European Journal of Operational Research* 195, 575–594.

Sahoo, B.K., Tone, K., 2009b. Radial and non-radial decompositions of profit change: With an application to Indian banking. *European Journal of Operational Research* 196, 1130–1146.

Seiford, L.M., Zhu, J., 2002. Modeling undesirable factors in efficiency evaluation. *European Journal of Operational Research* 142, 16–20.

Sengupta, J.K., 1999. A dynamic efficiency model using data envelopment analysis. *International Journal of Production Economics* 62, 209–218.

Sengupta, J.K., 2002. Model of hypercompetition. *International Journal of Systems Science* 33, 669–675.

Sengupta, J.K., 2003. *New Efficiency Theory: With Applications of Data Envelopment Analysis*. Springer-Verlag, Berlin.

Sengupta, J.K., 2004. *Competition and Growth: Innovations and Selection in Industry Evolution*. Palgrave Macmillan, London.

Sengupta, J.K., 2005a. Nonparametric efficiency analysis under uncertainty using data envelopment analysis. *International Journal of Production Economics* 95, 39–49.

Sengupta, J.K., 2005b. Data envelopment analysis with heterogeneous data: An application. *Journal of the Operational Research Society* 56, 676–686.

Sengupta, J.K., 2007. *Dynamics of Entry and Market Evolution*. Palgrave Macmillan, New York.

Sengupta, J.K., 2011. *Understanding Economic Growth: Modern Theory and Experience*. Springer, New York.

Sengupta, J.K., Neogi, C., 2009. *India's New Economy: Industry Efficiency and Growth*. Palgrave Macmillan, London.

Sengupta, J.K., Sahoo, B.K., 2006. *Efficiency Models in Data Envelopment Analysis: Techniques of Evaluation of Productivity of Firms in a Growing Economy*. Palgrave Macmillan, London.

Silva, E., Stefanou, S.E., 2003. Nonparametric dynamic production analysis and the theory of cost. *Journal of Productivity Analysis* 19, 5–32.

Silva, E., Stefanou, S.E., 2007. Nonparametric dynamic efficiency analysis. *American Journal of Agricultural Economics* 89, 398–419.

Solow, R.M., 1997. *Learning from Learning by Doing: Lessons for Economic Growth*. Stanford University Press, Stanford, CA.

Sueyoshi, T., 1999. DEA duality on returns to scale (RTS) in production and cost analyses: An occurrence of multiple solutions and differences between production-based and cost-based RTS estimates. *Management Science* 45, 1593–1608.

Sueyoshi, T., Sekitani, K., 2005. Returns to scale in dynamic DEA. *European Journal of Operational Research* 161, 536–544.

Tone, K., Sahoo, B.K., 2003. Scale, indivisibilities and production function in data envelopment analysis. *International Journal of Production Economics* 84, 165–192.

Tone, K., Sahoo, B.K., 2004. Degree of scale economies and congestion: A unified DEA approach. *European Journal of Operational Research* 158, 755–772.

Tone, K., Sahoo, B.K., 2005. Cost efficiency and returns to scale in life insurance corporation of India using data envelopment analysis. *Socio-Economic Planning Sciences* 39, 261–285.

Tone, K., Sahoo, B.K., 2006. Re-examining scale elasticity in DEA. *Annals of Operations Research* 145, 169–187.